GUROBI
OPTIMIZATION

Using Multiple Machines to Solve Models Faster with Gurobi 6.0

Distributed Algorithms in Gurobi 6.0

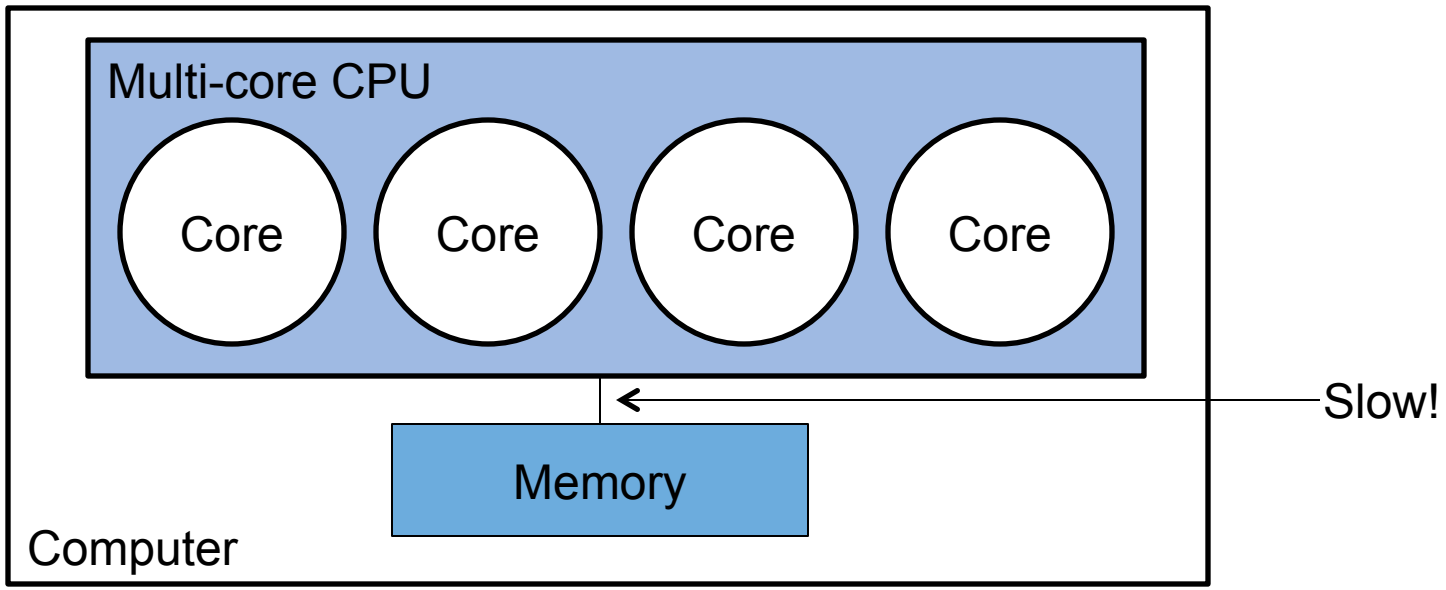
- Gurobi 6.0 includes 3 distributed algorithms
  - Distributed concurrent
    - LP (new in 6.0)
    - MIP
  - Distributed MIP (new in 6.0)
  - Distributed tuning

Distributed Tuning

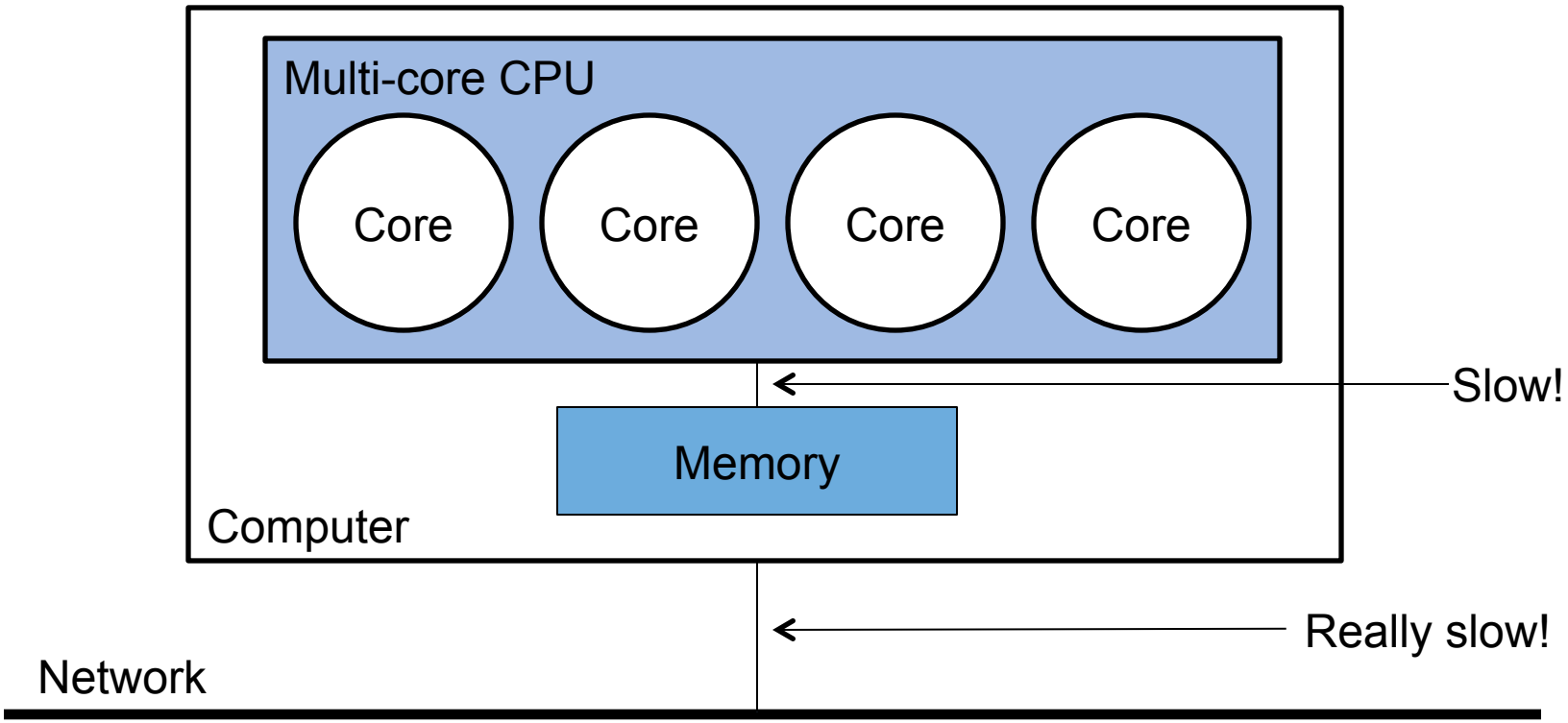
- Tuning:
  - MIP has lots of parameters
  - Tuning performs test runs to find better settings
- Obvious candidate for parallelism
- Distributed tuning a clear win
  - 10X faster on 10 machines
- Hard to go back once you have tried it

Parallel Optimization Challenges

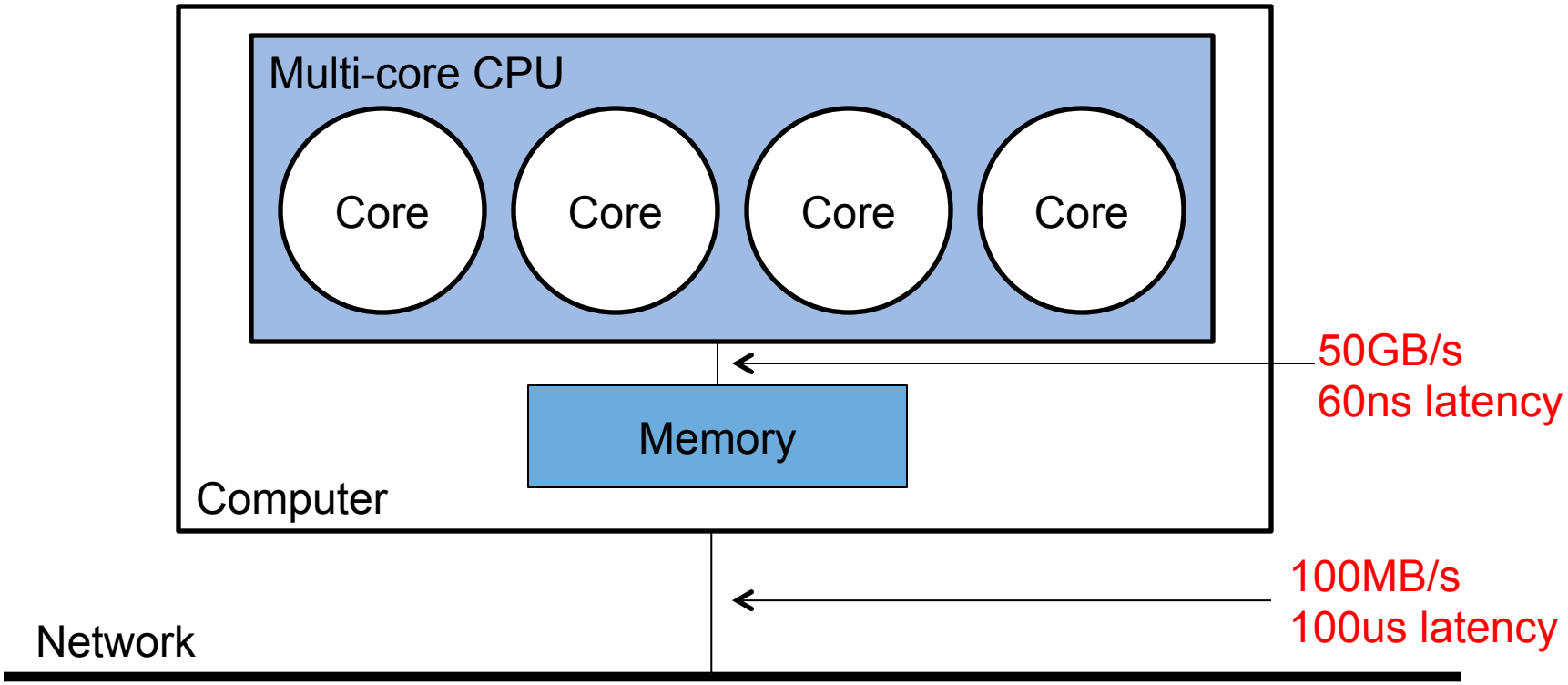
Multi-Core Machines



Distributed Computing



How Slow Is Communication?



- Network is ~1000X slower than memory
  - Faster on a supercomputer (but still slow)

Concurrent Optimization

Concurrent Optimization

- Run different algorithms/strategies on different machines/cores
  - First one that finishes wins
- Nearly ideal for distributed optimization
  - Communication:
    - Send model to each machine
    - Winner sends solution back
- Concurrent LP:
  - Different algorithms:
    - Primal simplex/dual simplex/barrier
- Concurrent MIP:
  - Different strategies
  - Default: change random number seed

MIPLIB 2010 Testset

- MIPLIB 2010 test set…
  - Set of 361 mixed-integer programming models
  - Collected by academic/industrial committee
- MIPLIB 2010 benchmark test set…
  - Subset of the full set – 87 of the 361 models
    - Those that were solvable by 2010 codes
    - (Solvable set now includes 206 of the 361 models)
  - A few notes:
    - Definitely not intended as a high-performance computing test set
      - More than 2/3 solve in less than 100s
      - 8 models solve at the root node
      - ~1/3 solve in less than 1000 nodes

Distributed Concurrent MIP

- Results on MIPLIB benchmark set (>1.00X means concurrent MIP is faster):
  - 4 machines vs 1 machine:

Runtime	Wins	Losses	Speedup
>1s	38	20	1.26X
>100s	17	3	1.50X

  - 16 machines vs 1 machine:

Runtime	Wins	Losses	Speedup
>1s	54	19	1.40X
>100s	26	1	2.00X

Mechanics

Gurobi Remote Services

- Install Gurobi Remote Services on worker machines
  - No Gurobi license required
  - Machine listens for Distributed Worker requests
- Set a few parameters on manager
  - `ConcurrentJobs=4`
  - `WorkerPool=machine1,machine2,machine3,machine4`
  - No other code changes required
- Manager must be licensed to use distributed algorithms
  - Gurobi Distributed Add-On
    - $6K add-on for named-user license, $12K for server license
    - Enables up to 100 workers

Integral Part of Product

- Built on top of Gurobi Compute Server
  - Only 1500 lines of C code specific to concurrent/distributed MIP
- Built into the product
  - No special binaries involved
- Bottom line:
  - Changes to MIP solver automatically apply to distributed code too
  - Distributed MIP will evolve with regular MIP

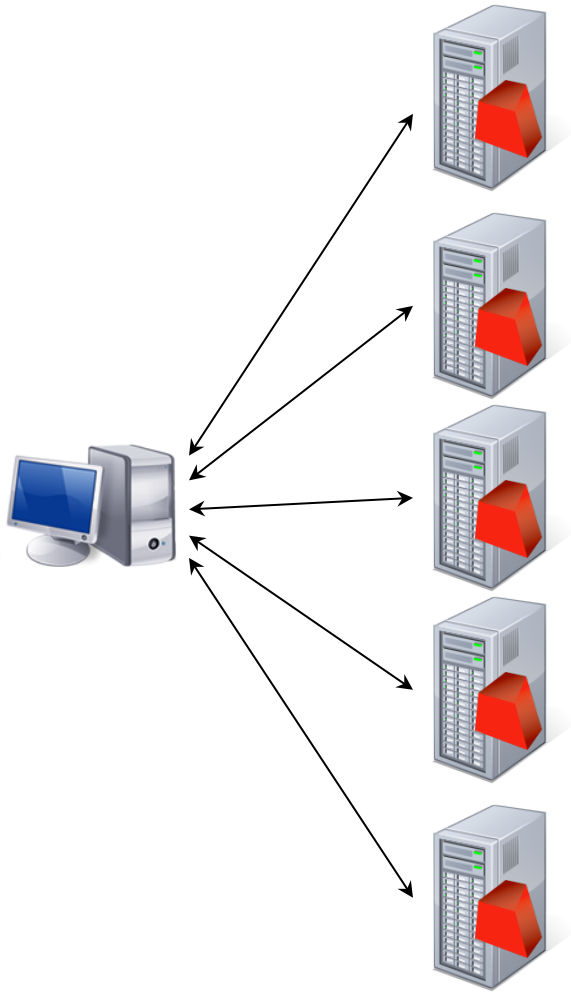
Distributed Concurrent

- Easy to choose your own settings:
  - Example – 2 concurrent MIP solves:
    - Aggressive cuts on one machine
    - Aggressive heuristics on second machine

Distributed MIP

Distributed MIP Architecture

- Manager-worker paradigm
- Manager
  - Send model to all workers
  - Track dual bound and worker node counts
  - Rebalance search tree to put useful load on all workers
  - Distribute feasible solutions
- Workers
  - Solve MIP nodes
  - Report status and feasible solutions
- Deterministic synchronization



Distributed MIP Phases

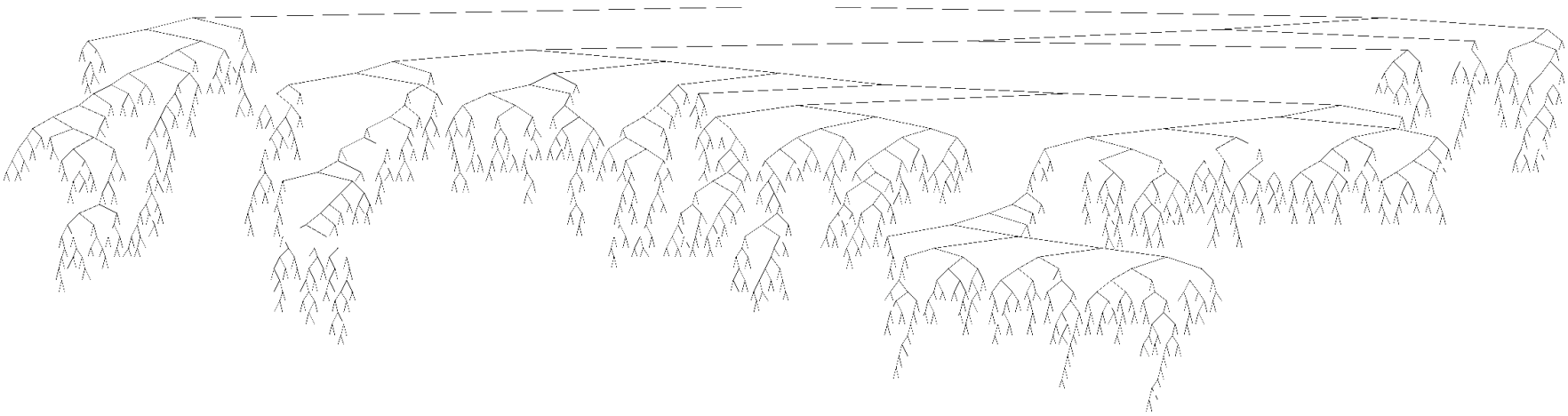
- Racing ramp-up phase
  - Distributed concurrent MIP
    - Solve same problem individually on each worker, using different parameter settings
    - Stop when problem is solved or “enough” nodes are explored
    - Choose a “winner” – worker that made the most progress
- Main phase
  - Discard all worker trees except the winner's
  - Collect active nodes from winner, distribute them among now idle workers
  - Periodically synchronize to rebalance load

Distributed MIP Performance

- Key questions:
  - Is there enough work to keep workers busy?
    - Total work
    - Shape of MIP search tree
  - How expensive is it to move work between machines?
    - Typical size of a MIP subtree

Good Cases for Distributed MIP

- Large search trees
- Well-balanced search trees
  - Many nodes in frontier lead to large sub-trees

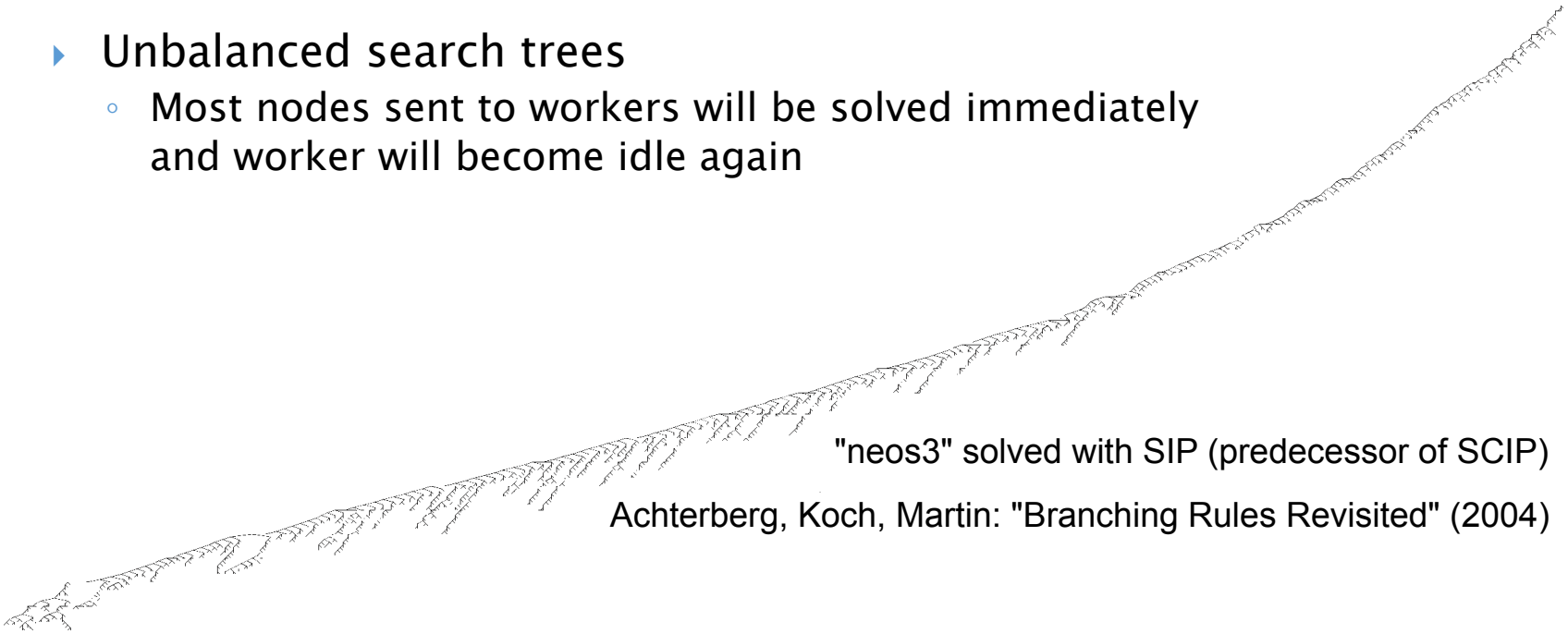


"vpm2" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)

Bad Cases for Distributed MIP

- Easy problems
  - Why bother with heavy machinery?
- Small search trees
  - Nothing to gain from parallelism
- Unbalanced search trees
  - Most nodes sent to workers will be solved immediately and worker will become idle again



"neos3" solved with SIP (predecessor of SCIP)

Achterberg, Koch, Martin: "Branching Rules Revisited" (2004)

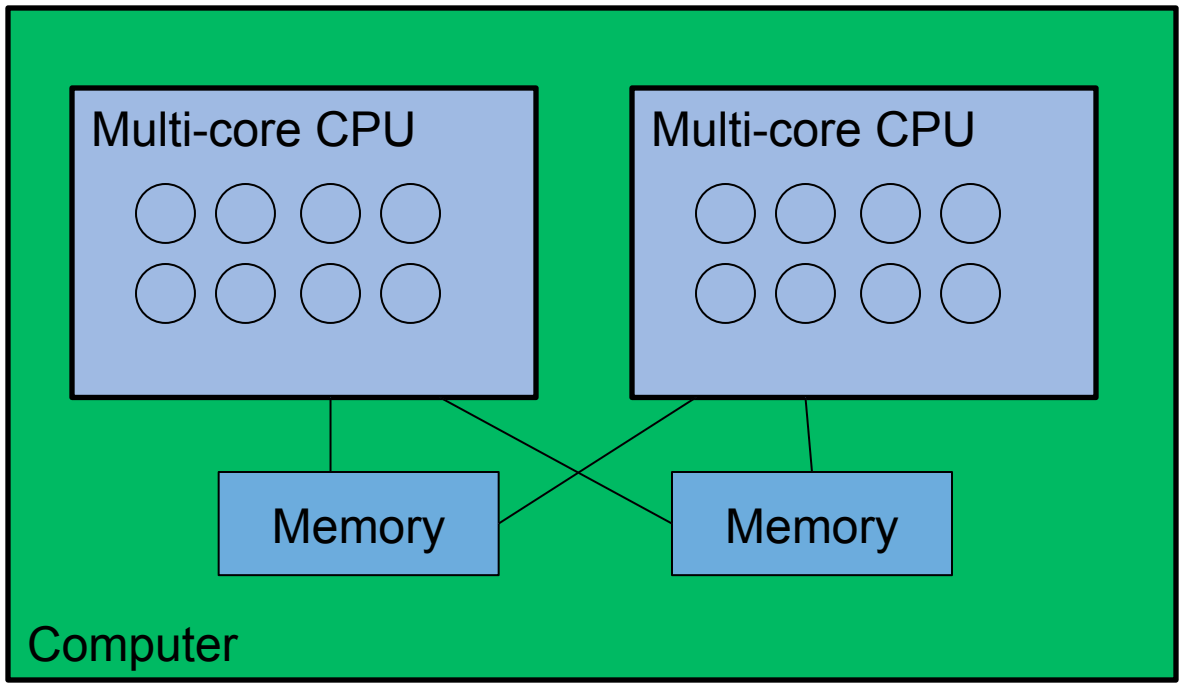
Performance

Three Views of 16 Cores

- Consider three different tests, all using 16 cores:
  - On a 16-core machine:
    - Run the standard parallel code on all 16 cores
    - Run the distributed code on four 4-core subsets
  - On four 4-way machines:
    - Run the distributed code
- Which gives the best results?

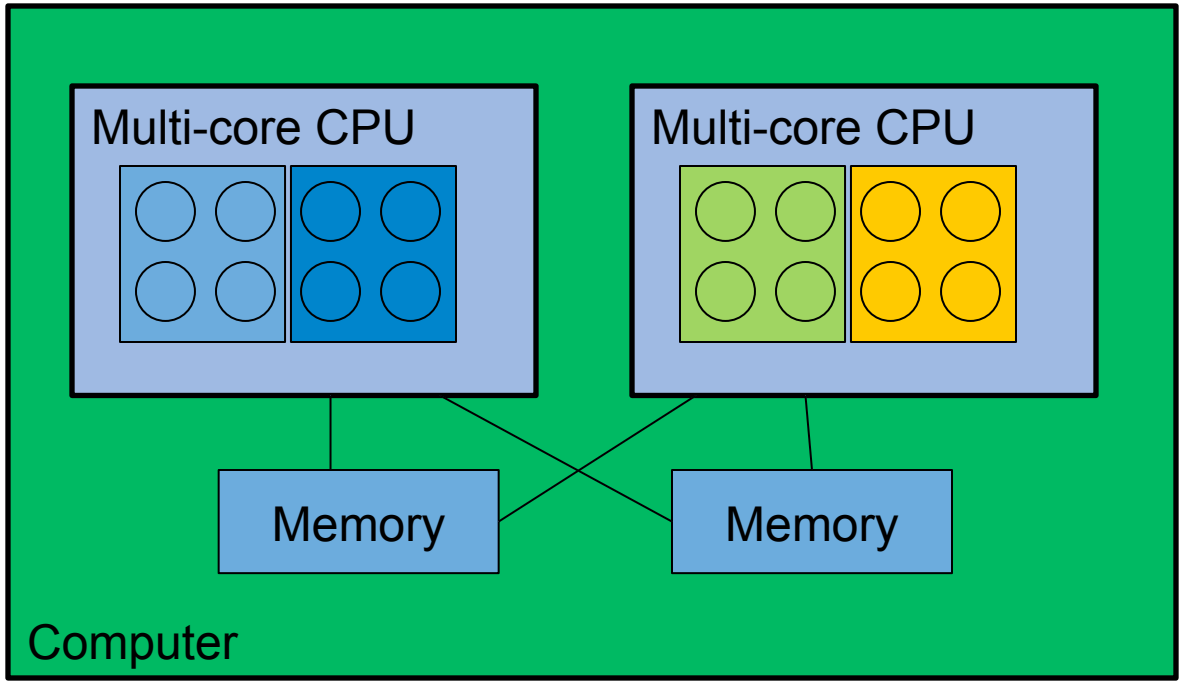
Parallel MIP on 1 Machine

- Use one 16-core machine:



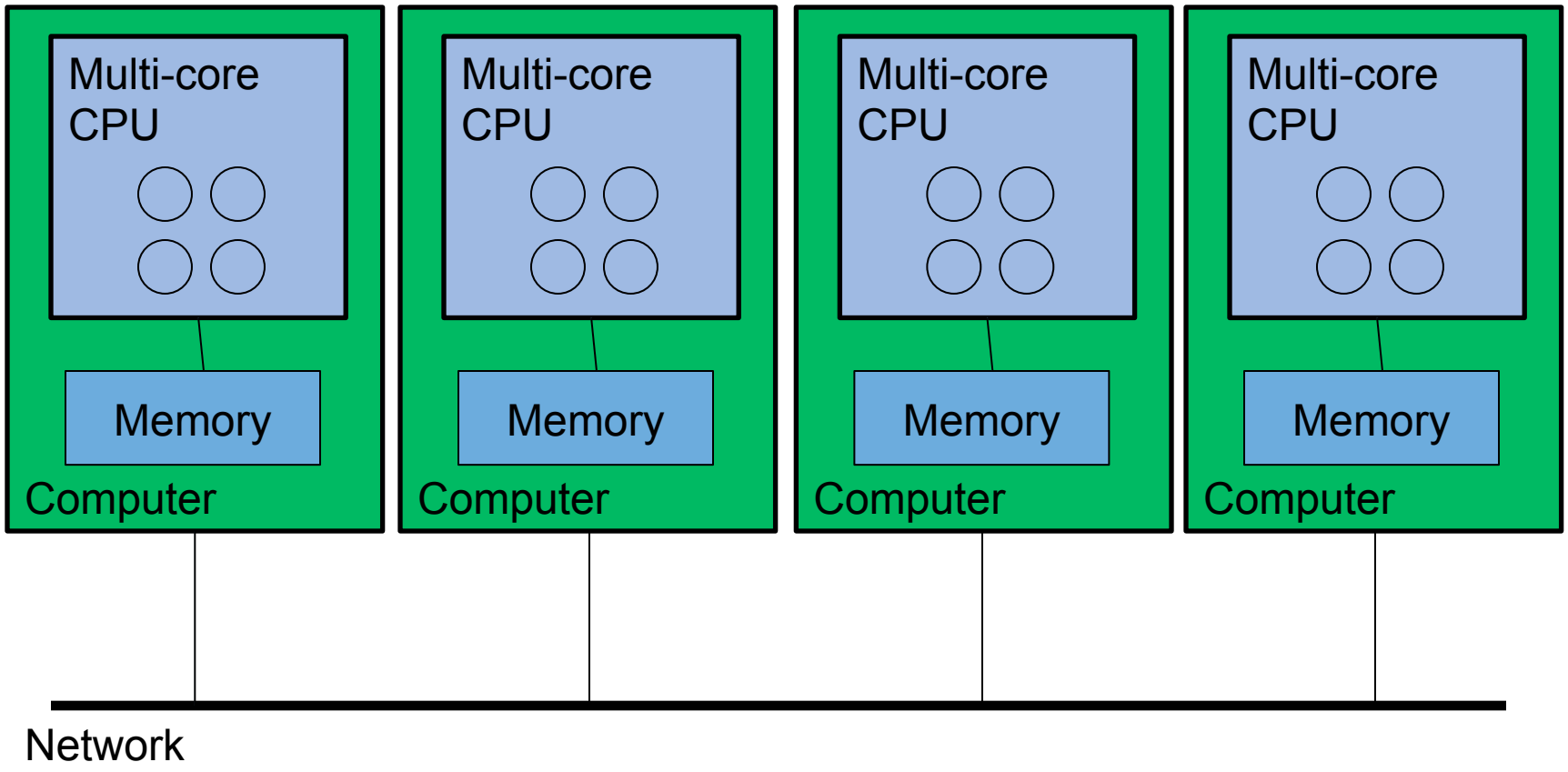
Distributed MIP on 1 machine

- Treat one 16-core machine as four 4-core machines:



Distributed MIP on 4 machines

- Use four 4-core machines



Performance Results

- Using one 16-core machine (MIPLIB 2010, baseline is 4-core run on the same machine)…

Config	>1s	>100s
One 16-core	1.57X	2.00X
Four 4-core	1.26X	1.82X

- Better to run one-machine algorithm on 16 cores than treat the machine as four 4-core machines
  - Degradation isn't large, though

Performance Results

- Comparing one 16-core machine against four 4-core machines (MIPLIB 2010, baseline is single-machine, 4-core run)…

Config	>1s	>100s
One 16-core machine	1.57X	2.00X
Four 4-core machines	1.43X	2.09X

- Given a choice…
  - Comparable mean speedups
  - Other factors…
    - Cost: four 4-core machines are much cheaper
    - Admin: more work to admin 4 machines

Distributed Algorithms in 6.0

- MIPLIB 2010 benchmark set
  - Intel Xeon E3-1240v3 (4-core) CPU
  - Compare against 'standard' code on 1 machine

Machines	>1s			>100s		
	Wins	Losses	Speedup	Wins	Losses	Speedup
2	40	16	1.14X	20	7	1.27X
4	50	17	1.43X	25	2	2.09X
8	53	19	1.53X	25	2	2.87X
16	52	25	1.58X	25	3	3.15X

Some Big Wins

- Model *seymour*
  - Hard set covering model from MIPLIB 2010
  - 4944 constraints, 1372 (binary) variables, 33K non-zeroes

Machines	Nodes	Time (s)	Speedup
1	476,642	9,267s	-
16	1,314,062	1,015s	9.1X
32	1,321,048	633s	14.6X

Some Big Wins

- Model *a1c1s1*
  - Hard lot sizing model from MIPLIB 2010
  - 3312 constraints, 3648 variables (192 binary), 10K non-zeroes

Machines	Nodes	Time (s)	Speedup
1	3,510,833	17,299s	-
49	9,761,505	1,299s	13.3X

Distributed Concurrent Versus Distributed MIP

- MIPLIB 2010 benchmark set (versus 1 machine run):
  - >1s

Machines	Concurrent	Distributed
4	1.26X	1.43X
16	1.40X	1.58X

  - >100s

Machines	Concurrent	Distributed
4	1.50X	2.09X
16	2.00X	3.15X

Gurobi Distributed MIP

- Makes huge improvements in performance possible
- Mean performance improvements are significant but not huge
  - Much better than distributed concurrent
  - As effective as adding more cores to one box
- Effectively exploiting parallelism remains:
  - A difficult problem
  - A focus of ours

Thank You

- New academic users should visit our Academic Licensing page to get free academic versions of Gurobi
  - http://www.gurobi.com/products/licensing-and-pricing/academic-licensing
- New commercial users should request a free evaluation version of Gurobi either directly from your account rep or by emailing sales@gurobi.com.
- If you have any general questions, please feel free to email info@gurobi.com.