Optimization Application Demo

Cutting Stock Problem with Multiple Master Rolls

Introduction

Cutting stock problems can be classified in several ways. One way is the dimensionality of the cutting: a one-dimensional (1D) cutting stock problem occurs at a paper mill where large paper rolls are cut into smaller width paper rolls; other industrial applications of 1D problems occur when cutting pipes, cables, and steel bars. Two-dimensional (2D) problems are encountered in furniture, clothing, and glass production. Not many three-dimensional (3D) applications involving cutting are known; however, the closely related 3D packing problem has many industrial applications, such as packing objects into shipping containers.

Cutting stock problem with multiple master rolls

Let's look at a simple example of a paper mill that needs to minimize operating costs while facing certain constraints. The mill supplies paper rolls or "final rolls" to customers that are cut from several master rolls of different widths. The width of a master roll defines a master roll type.

The need is to generate a master roll cutting plan that minimizes the cost of cutting and procuring master rolls consumed to satisfy all the demand of the final rolls.

For each master roll type, there is a per unit cutting cost and an initial inventory available at the beginning of the planning horizon. Extra master rolls of each type can be bought at a procurement cost. There is aggregate demand for smaller rolls (final rolls) cut from a master roll. The width of a final roll defines a demand type. Master rolls are then cut into final rolls in order to meet demand.

We assume that there is a single machine that cuts the different types of master rolls to satisfy the aggregate demand of final rolls. The machine has a specific number of usable knives to cut the master roll. The number of knives in the machine limits the possible cutting configurations.

After a master roll is cut, the left-over (spare) roll may be re-usable if the spare roll width is larger than a specified threshold width.

Master Roll Type	Width (Inches)	MRCost	CutCost	Inventory
m1	5"	$10	$1	3
m2	10"	$20	$2	2

Table 1: Parameters of master roll types.

Example of a cutting stock problem with multiple master rolls

To ground the ideas behind the cutting stock problem with multiple rolls, let's consider the following example described in table 1. The column "Master roll type" has the ID of each master roll type. The column "Width" has the width of each master roll type. The column "MRCost" has the procurement cost of each master roll type. The column "CutCost" has the cutting cost of each master roll type. The column "inventory" has the initial inventory of each master roll type. For example, the master roll of type m1 has a width of 5'', its procurement cost is $10, its cutting cost $1, and there are 3 units of initial inventory.

There are six types of final rolls, the parameters of these final rolls are described in table 2. The column "Final roll type" has the ID of each type of final roll. The column "Width" has the width of each type of final roll. The column "Demand" has the aggregate demand of each type of final roll. For example, the final roll of type d1 has a width of 3'', and its aggregate demand is 5 units.

Final Roll Type	Width (inches)	Demand
d1	3"	5
d2	2"	7
d3	1"	11
d4	8"	8
d5	7"	7
d6	6"	13

Table 2: Parameters of final rolls types.

Key assumptions for this demo

- We assume a single cutting machine
- We assume a single planning period
- Zero inventory carrying cost of a master roll
- Zero procurement lead time
- Zero setup cost when adjusting the knives of the cutting machine from one cutting configuration to another
- Unlimited availability of master rolls to buy
- Aggregate demand of final rolls – no order visibility

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Optimization Process

The optimization approach to tackling the cutting stock problem entails two steps:

1. The algorithm to generate cutting patterns: For a given set of widths of master rolls and set of widths of final rolls and aggregate demand for each final roll type, generate all the possible cutting patterns of each master roll type. Each cutting pattern consumes a master roll –i.e. the cutting pattern configuration minimizes unused material.
2. The model for the cutting stock problem with multiple master rolls: For a given initial inventory of each maste roll type, aggregate demand of each final roll type and all feasible cutting patterns of each master roll type to satisfy all demand of final rolls; find the number of times that a cutting pattern of each master roll type will be used to satisfy all aggregate demand of final rolls in order to minimize procurement costs and cutting costs of all master rolls consumed.

Output Reports

The main output reports of this cutting stock demo are:

- Cutting operations
- Buying order

Consider the example described in Tables 1 and 2.

Assume that the minimum width of a master roll type to be re-usable (spare threshold) is 2''. Also, assume that there is no restriction on the maximum number of knives that can be used. The output report for the "cutting operations" is presented in figure 1.

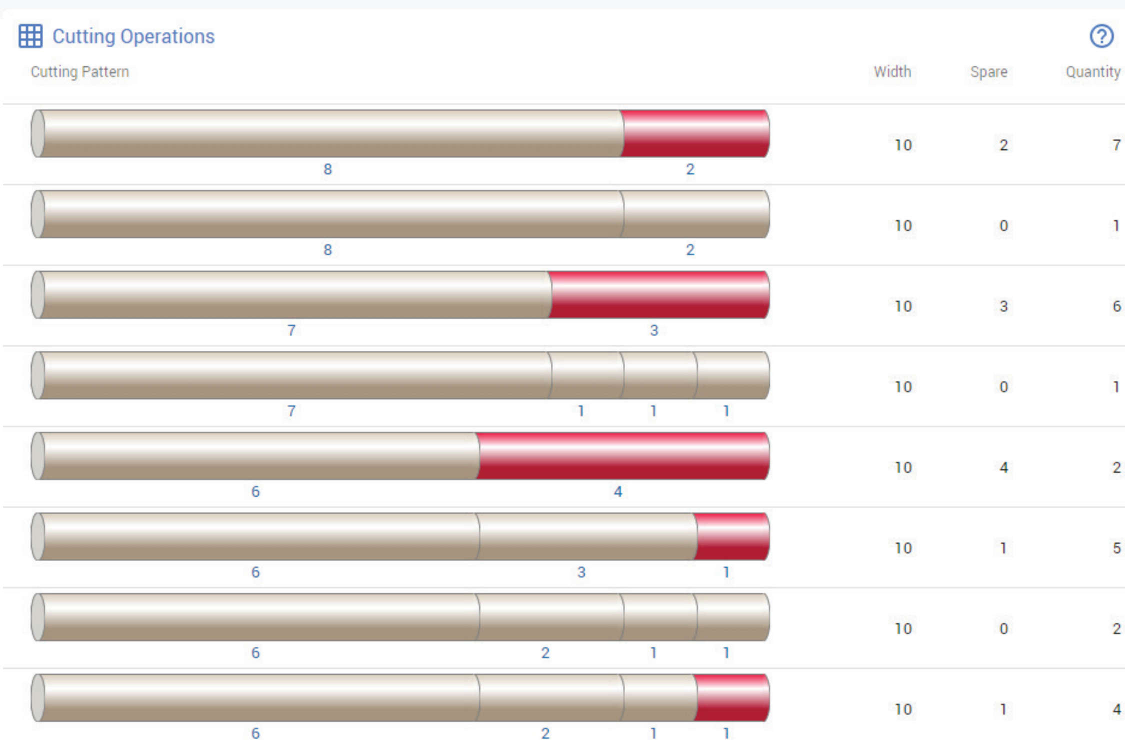

Figure 1: Cutting Operations output report.

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Each row in this output report has a cutting pattern for a master roll of a given width (in inches). The spare is the number of unused inches of the master roll and quantity is the number of master rolls of this width that are cut using the cutting pattern.

Notice that all the master rolls consumed to satisfy demand of final rolls are of a 10'' width.

For example, the first row has a cutting pattern where one final roll of width 8'' is cut from a master roll of width 10'' leaving 2 '' of unused material (in red), and 7 master rolls of this width are cut using this cutting pattern.

Another example is to consider the seventh row where the cutting pattern is to cut one final roll of width 6'', one final roll of width 2'', and two final rolls of width 1'' from a master roll of width 10''. In this case there is no unused material and 2 master rolls of this width are cut using this cutting pattern.

The output report of "buying order" is presented in figure 2.

Buying Order

Width	Quantity
10	26

Figure 2: Buying Order output report.

In this report, we have the width and quantity of master rolls procured. In this example, we procure 26 master rolls of width 10''.

Key Performance Indicators (KPIs)

For the cutting stock problem with multiple master rolls, it is very important to understand how efficiently the master rolls of different types have been used to fill the demand of final rolls. Conversely, it is also important to understand how much unused width of master rolls remains after cutting. The unused width of these master rolls might be big enough to fill demand of various small width final rolls, therefore they can be re-used to fill future demand. However, there might be other master rolls with unused width too small to be re-used; these master rolls are then scrapped. In addition, it is also important to develop metrics for consumption of master rolls inventories on-hand and procured. For these purposes, we have developed the following performance metrics and reports.

Consumption Efficiency

This metric measures how well the cutting patterns consume the master rolls to satisfy final roll aggregate demand. Since we know that all the demand of the final rolls is satisfied- this fact can be used to determine the efficiency metric. Therefore, this metric can be computed by calculating the width of the total demand of final rolls, and then dividing it by the total width of master rolls consumed to fill all the aggregate demand of final rolls.

To give an idea about how the consumption efficiency metric can be calculated, let's consider the following simple example. From the last row in the cutting pattern output report of figure 1, we have a cutting pattern where we cut from a master roll of width 10'', one final roll of width 6'', one final roll of width 2'', one final roll of width 1'', and we have unused material of width 1'' (See figure 3)

Figure 3: cutting pattern to cut one master roll of width 10''.

The total width of final rolls cut from this master roll is: 6'' + 2'' + 1'' = 9''. The width of the master roll is 10''. Therefore, the consumption efficiency is

$$9''/10'' = 0.9$$

Consumption inefficiency

This metric measures how much of the master roll cut was not consumed to satisfy demand of final rolls. This metric can be computed by calculating the total width of the unused material of master rolls consumed to satisfy demand of final rolls, and then divided by the total width of master rolls consumed to fill all the aggregate demand of the final rolls. However, a simpler way to compute the consumption inefficiency is to subtract the consumption efficiency from 1.

For example, consider the cutting pattern in figure 3. In this case, we have 1'' of unused material from the master roll, and the width of the master roll is 10''. Consequently, the consumption inefficiency is

$$1''/10'' = 0.1$$

Notice that the consumption inefficiency can be computed as 1 minus the consumption efficiency:

$$1 - 0.9' = 0.1$$

As we have discussed, a threshold quantity can be defined to determine if the unused master roll material can be re-used (spare) or otherwise scrapped. That is, if the width of the unused material of a master roll consumed to satisfy demand is greater or equal to the threshold value, then the spare master roll can be re-used to satisfy future final roll demand. Otherwise, the unused material of the master roll is scrapped.

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Reusable Spare

To compute this metric, we only consider the width of the unused material of master rolls that have a width equal or larger than threshold value.

Scrap

To compute this metric, we only consider the width of the unused material of master rolls that have a width less than threshold value.

Notice that the overall master rolls spare metric plus the overall master rolls scrap metric is equal to the overall inefficiency metric of master rolls.

From the cutting pattern output report of figure 1, let's consider the following two cutting patterns

Figure 4: Two cutting patterns for a master roll of width 10''.

Let's assume that the spare threshold is 2''. Notice that the unused material of the first cutting pattern, in figure 4, is 2'' which is equal to the spare threshold. Also, observe that the unused material of the second cutting pattern, in figure 4, is 1'' which is less than the spare threshold. In this example, we considered two master rolls, each of a 10'' width.

Hence, the reusable spare metric can be computed as follows:

$$2''/20'' = 0.1$$

The scrap metric can be computed as follows:

$$1''/10'' = 0.05$$

Let's compute now the inefficiency metric.

$$(2'' + 1'')/20'' = 0.15$$

Notice that the inefficiency metric is equal to the reusable spare metric plus scrap metric:

$$0.1 + 0.05 = 0.15$$

Capacity utilization

This metric measures how much of the inventory on-hand and procured is utilized by the master rolls consumed to satisfy demand. This metric can be computed by calculating the total width of master rolls consumed to fill demand, divided by the total width of master rolls in inventory and procured. Notice that for this metric the unused material of a consumed master roll is considered in favor of the metric.

To illustrate how to compute the capacity utilization metric, let's consider the example in figure 4, assuming that we have one master roll of width 10'' as initial inventory and that we have procured another of the same width. Therefore, the capacity utilization metric value is

$$c = 10'' * 2/(10 + 10'') = 1$$

Inventory utilization

This metric measures how much of the width of the inventory on-hand and procured is utilized to satisfy demand. This metric can be computed by calculating the width of the total demand of final rolls divided by the total width of master rolls in inventory and procured. Observe that this metric does not consider the unused material of a consumed master roll.

Consider the same example as in the capacity utilization section. Notice that the final roll demand satisfied by using the first cutting pattern in figure 4 is 8'', and that the final roll demand satisfied by using the second cutting pattern in figure 4 is (6'' + 2'' + 1'') = 9''.

Consequently, the inventory utilization metric value is:

$$i = [8'' + (6'' + 2'' + 1'')]/(10'' + 10'') = 0.85$$

Conclusion

The paper provides an overview of the Gurobi Optimization Application Demo by illustrating one related to a cutting stock problem with multiple master rolls.

The optimization process to solve the cutting stock problem entails two fundamental steps: an algorithm to generate all the possible cutting patterns and a Mixed Integer Programming (MIP) formulation to solve the problem considering all the possible cutting patterns.

The optimal solution of the cutting stock problem entails the master rolls' cutting patterns and the number of master rolls to purchase in order to minimize the total cost of cutting and procuring new master rolls, while satisfying all the aggregate demand of final rolls.

KPI metrics related to the operational efficiency of any solution of the cutting stock problem are explained in detail.